Exam Lie Groups in Physics

Date	February 2, 2018
Room	5419.0013
Time	18:30 - 21:30
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- Raise your hand for more paper
- You are not allowed to use the lecture notes, nor other notes or books
- The weights of the **four** problems are given below
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Weighting

1a)	6	2a)	8	3a)	8	4a)	6
1b)	6	2b)	8	3b)	6	4b)	6
1c)	6	2c)	8	3c)	6	4c)	6
1d)	4			3d)	6		

$$\text{Result } = \frac{\sum \text{points}}{10} + 1$$

Problem 1
Consider the Lie group $SU(3)$ of unitary 3×3 matrices with determinant 1.

(a) Derive the dimension of $SU(3)$.

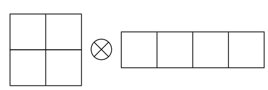
(b) Indicate a way in which $SU(2)$ can be viewed as a subgroup of $SU(3)$ and explain why $SU(3)/SU(2)$ does not form a group.

(c) Determine the center of $SU(3)$ (you may assume that the defining rep is an irrep).

(d) Give an example from physics where $SU(3)$ plays a role.

Problem 2
Consider the Lie algebra $su(n)$ of the Lie group $SU(n)$ of unitary $n \times n$ matrices with determinant equal to 1.

(a) Decompose the following direct product of irreps of the Lie algebra $su(n)$



into a direct sum of irreps of $su(n)$, in other words, determine its Clebsch-Gordan series.

(b) Write down the dimensions of the irreps appearing in the obtained decomposition for $su(3)$ and $su(4)$. Indicate the complex conjugate and inequivalent irreps whenever appropriate.

(c) Consider for $su(3)$ the complex conjugate of the above direct product in terms of Young tableaux, decompose it into irreps and compare the answer to the one obtained in part (b).

Problem 3

Consider the group $O(1,1)$ defined by 2×2 real matrices O satisfying

$$O^T = gO^{-1}g^{-1} \quad \text{with} \quad g = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(a) Write down the general form of elements O in $O(1,1)$ and show that such matrices form a non-compact non-Abelian group. Suggestion: first consider the determinant of the elements O.

(b) Specify the connected components of $O(1,1)$ and show that they form cosets of the connected subgroup. Describe the corresponding factor group.

(c) Show whether the defining representation is irreducible.

(d) Write down the corresponding representation of the Lie algebra of $O(1,1)$ and show whether it is an irrep of the Lie algebra.

Problem 4

Consider the Poincaré group P consisting of elements $(t|\Lambda)$, where t represents a translation in four-dimensional Minkowski space and Λ represents a Lorentz transformation.

(a) Show that the composition law of the Poincaré group is given by:

$$(t_1|\Lambda_1)(t_2|\Lambda_2) = (\Lambda_1 t_2 + t_1|\Lambda_1\Lambda_2).$$

(b) Show that every element can be *uniquely* written as

$$(t|\Lambda) = (t|\mathbf{1})(0|\Lambda).$$

The translations form a subgroup T of P and the Lorentz transformations form a subgroup L of P.

(c) Show that $P/T \cong L$ and that P/L does not form a group.